Unit 5: Special probability distributions

Two of the most widely used discrete probability distributions are the binomial and Poisson.

The binomial distribution

The binomial probability mass function (equation 6) provides the probability that *x* successes will occur in *n* trials of a binomial experiment.

$$f(x) = \binom{n}{x} p^x (1-p)^{(n-x)} \qquad (6)$$

A binomial experiment has four properties: (1) it consists of a sequence of *n* identical trials; (2) two outcomes, success or failure, are possible on each trial; (3) the probability of success on any trial, denoted *p*, does not change from trial to trial; and (4) the trials are independent. For instance, suppose that it is known that 10 percent of the owners of two-year old automobiles have had problems with their automobile's electrical system. To compute the probability of finding exactly 2 owners that have had electrical system problems out of a group of 10 owners, the binomial probability mass function can be used by setting $n = 10$, $x = 2$, and $p = 0.1$ in equation 6; for this case, the probability is 0.1937.

The Poisson distribution

The Poisson probability distribution is often used as a model of the number of arrivals at a facility within a given period of time. For instance, a random variable might be defined as the number of telephone calls coming into an airline reservation

system during a period of 15 minutes. If the mean number of arrivals during a 15-minute interval is known, the Poisson probability mass function given by equation 7 can be used to compute the probability of *x* arrivals.

$$f(x) = \frac{\mu^x e^{-\mu}}{x!} \qquad (7)$$

For example, suppose that the mean number of calls arriving in a 15-minute period is 10. To compute the probability that 5 calls come in within the next 15 minutes, μ = 10 and *x* = 5 are substituted in equation 7, giving a probability of 0.0378.

The normal distribution

The most widely used continuous probability distribution in statistics is the normal probability distribution. The graph corresponding to a normal probability density function with a mean of μ = 50 and a standard deviation of σ = 5 is shown in Figure 3. Like all normal distribution graphs, it is a bell-shaped curve. Probabilities for the normal probability distribution can be computed using statistical tables for the standard normal probability distribution, which is a normal probability distribution with a mean of zero and a standard deviation of one. A simple mathematical formula is used to convert any value from a normal probability distribution with mean μ and a standard deviation σ into a corresponding value for a standard normal distribution. The tables for the standard normal distribution are then used to compute the appropriate probabilities.

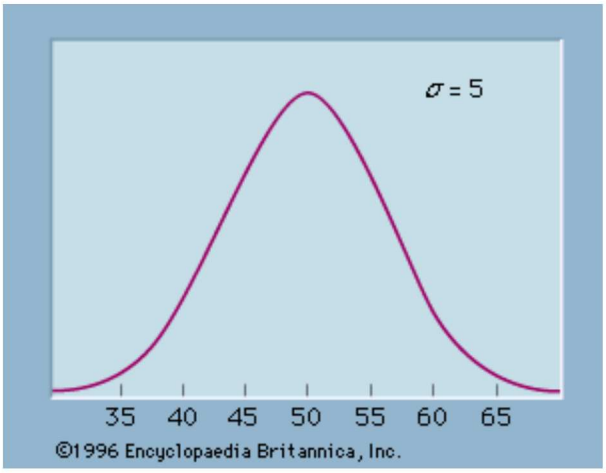

Figure 3: A normal probability distribution with a mean (μ) of 50 and a standard deviation (σ) of 5.

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There are many other discrete and continuous probability distributions. Other widely used discrete distributions include the geometric, the hypergeometric, and the negative binomial; other commonly used continuous distributions include the uniform, exponential, gamma, chi-square, beta, *t*, and F.

Comprehension Exercises

Choose a, b, c or d which best completes each item.

1) The number of arrivals at a bank within a given period of time can be modeled by the _____ probability distribution.

a) binomial b) geometric c) Poisson d) hypergeometric

2) The _____ probability distribution is the most widely used continuous probability distribution in statistics.

a) uniform b) exponential c) gamma d) normal

3) A simple mathematical formula is used to _____ any value from a normal probability distribution with mean μ and a standard deviation σ into a corresponding value for a standard normal distribution.

a) convert b) transform c) change d) make

4) The binomial, Poisson, geometric, hypergeometric and negative binomia are widely used _____ probability distributions.

a) discrete b) continuous c) bell-shaped d) identical

Words to Learn

Find the Persian equivalents of the following terms and expressions.

binomial	chi-square	arrival	out of
Poisson	beta distribution	period of time	widely
geometric	*t* distribution	telephone calls	appropriate
hypergeometric	F distribution	experiment	identical
negative binomial	model	bell-shaped	property
uniform	success	curve	exactly
normal	failure	mathematical	convert
exponential	trial	statistical tables	airline
gamma distribution	substitute	standard normal	reservation