احتمال پیشرفته	
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هفته‌ی سوم - جلسه‌ی ششم

حل برخی از تمرین‌های فصل‌های اول و دوم

Exercise 1.3.1. Suppose that $\Omega = \{1,2\}$, with $\mathbf{P}(\emptyset) = 0$ and $\mathbf{P}\{1,2\} = 1$. Suppose $\mathbf{P}\{1\} = \frac{1}{4}$. Prove that $\mathbf{P}$ is countably additive if and only if $\mathbf{P}\{2\} = \frac{3}{4}$.

Exercise 1.3.3. Suppose that $\Omega = \mathbf{N}$ is the set of positive integers, and $\mathbf{P}$ is defined for all $A \subseteq \Omega$ by $\mathbf{P}(A) = 0$ if A is finite, and $\mathbf{P}(A) = 1$ if A is infinite. Is $\mathbf{P}$ finitely additive?

Exercise 2.7.1. Let $\Omega = \{1,2,3,4\}$. Determine whether or not each of the following is a σ-algebra.
(a) $\mathcal{F}_1 = \Big\{\emptyset,\ \{1,2\},\ \{3,4\},\ \{1,2,3,4\}\Big\}$.
(b) $\mathcal{F}_2 = \Big\{\emptyset,\ \{3\},\ \{4\},\ \{1,2\},\ \{3,4\},\ \{1,2,3\},\ \{1,2,4\},\ \{1,2,3,4\}\Big\}$.
(c) $\mathcal{F}_3 = \Big\{\emptyset,\ \{1,2\},\ \{1,3\},\ \{1,4\},\ \{2,3\},\ \{2,4\},\ \{3,4\},\ \{1,2,3,4\}\Big\}$.

Exercise 2.7.5. Suppose that $\Omega = \mathbf{N}$ is the set of positive integers, and $\mathcal{F}$ is the set of all subsets A such that either A or A^C is finite, and $\mathbf{P}$ is defined by $\mathbf{P}(A) = 0$ if A is finite, and $\mathbf{P}(A) = 1$ if A^C is finite.
(a) Is $\mathcal{F}$ an algebra?
(b) Is $\mathcal{F}$ a σ-algebra?
(c) Is $\mathbf{P}$ finitely additive?
(d) Is $\mathbf{P}$ countably additive on $\mathcal{F}$, meaning that if $A_1, A_2, \ldots \in \mathcal{F}$ are disjoint, and if it happens that $\bigcup_n A_n \in \mathcal{F}$, then $\mathbf{P}(\bigcup_n A_n) = \sum_n \mathbf{P}(A_n)$?

Exercise 2.7.17. Let $\Omega = \{1,2\}$, and let $\mathcal{J}$ be the collection of all subsets of Ω, with $P(\emptyset) = 0$, $P(\Omega) = 1$, and $\mathbf{P}\{1\} = \mathbf{P}\{2\} = 1/3$.
(a) Verify that all assumptions of Theorem 2.3.1 <u>other</u> than (2.3.3) are satisfied.
(b) Verify that assumption (2.3.3) is <u>not</u> satisfied.
(c) Describe precisely the $\mathcal{M}$ and $\mathbf{P}^*$ that would result in this example from the modified version of Theorem 2.3.1 in Exercise 2.7.16(b).